Überreicht von den Verfassern

REPRESENTATION OF DEPTH-CONTOUR MAPS OF ARBITRARILY CURVED REFLECTION HORIZONS, INCLUDING REFRACTION OF RAYS, THREE-DIMENSIONAL CASE

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E. GRAESER, W. LODE, and G. POTT

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REPRESENTATION OF DEPTH-CONTOUR MAPS OF ARBITRARILY CURVED REFLECTION HORIZONS, INCLUDING REFRACTION OF RAYS, THREE-DIMENSIONAL CASE *

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ABSTRACT

A method of constructing depth contour maps of arbitrarily curved horizons obtained from seismic reflection surveys is discussed. This method takes into account three dimensional refraction, avoiding the construction of seismic cross sections of any kind. It requires little work even if refraction at several horizons is taken into account. The multiple layer problem is traced back to the single layer case. Discontinuities in velocity are also taken into account.

Under favourable circumstances a preliminary general picture of the position of reflection horizons may be obtained by considering one seismic reflection profile only. A popular method of constructing such a profile from the recorded normal reflection times (i.e. reflection times at zero shooting distance) is the so called tangent construction method which assumes (not always correctly) the reflected rays to be in the plane of the profile. After adequate terrain corrections, the normal reflection times recorded at the shot points are multiplied by the instantaneous velocity which is assumed to be known. Circles whose radii are the distances thus obtained, are then drawn around the shot points. The envelope of these circles is approximated by means of common tangents to neighbouring circles. It then represents the intersection of the plane of the profile with the reflecting boundary plane.

The three dimensional form of the ray paths is taken into account by considering several intersecting profiles. In simple cases the 'true dip' can be determined geometrically at the points of intersection of pairs of seismic profiles. However, in the strict mathematical sense this is possible only if the reflecting interface beneath the shot point can be assumed to be plane. This

*) Part of this paper was presented by W. Lode at the Tenth Meeting of the European Association of Exploration Geophysicists, held in Hamburg, 16-18 May 1956.

**) Institute of Mathematics of Goettingen University; research associate of *Prakla* G.m.b.H., Hannover.

***) *Prakla* G.m.b.H., Hannover.

special case has been treated in detail by J. Baumgarte (1955), who also suggests a method of construction. In practice this method is seldom applicable, as the construction involves a considerable expenditure of time. Besides in problems such as that shown in figure 1 it does not lead to a solution.

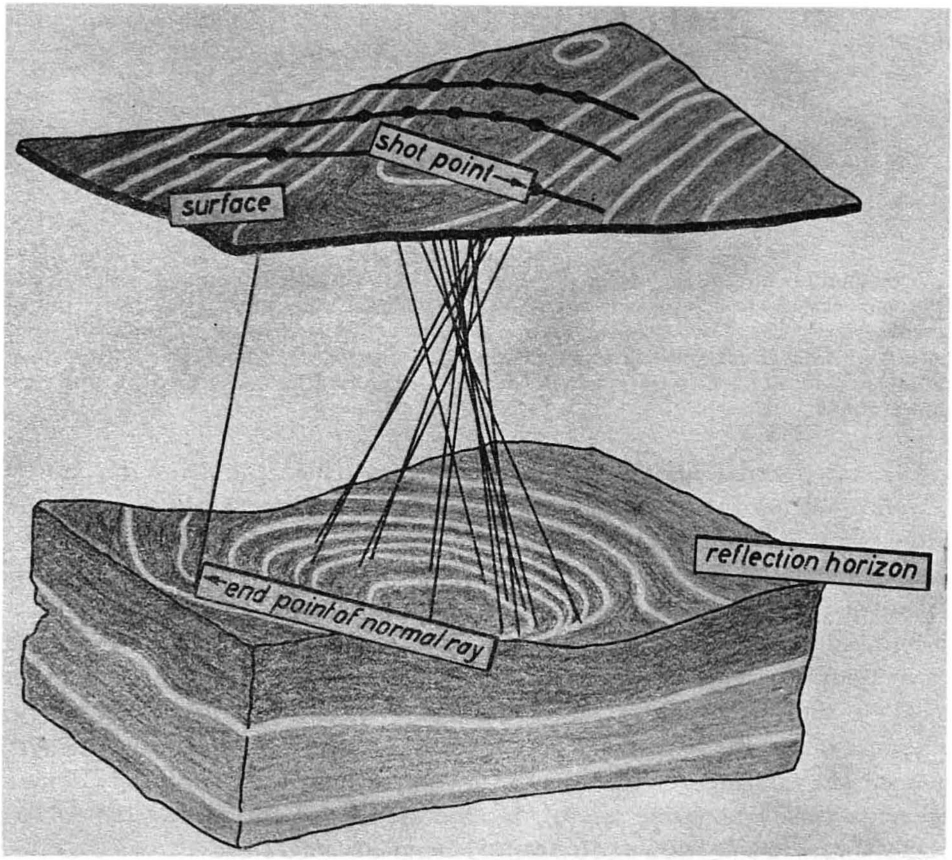

Fig. 1. Showing normal seismic paths in the single layer case.

Another method for the determination of interfaces with arbitrary curvature which has been given by M. Weber (1955), is an extension of the usual method for the construction of seismic horizons. This method is hardly applicable in routine work, since it requires seismometer spreads of considèrable length for horizons even at medium depths. Apart from this, it cannot be used when several reflecting horizons are present.

The present study differs from the above mentioned methods in that it presents an exact method for the construction of depth contour maps of arbitrarily curved horizons from observed normal reflection times. It takes into account the refraction of the rays in space when the overburden consists of layers having different velocities.

The method does not require the construction of seismic cross sections, but allows depth contour maps to be constructed rapidly and directly from normal depth* maps.

Since the strike and dip of the reflecting interface need not be taken into consideration when laying out the profiles on the earth's surface, they can be arranged along any kind of roads or field tracks in order to avoid damage to crops etc., unless subsurface conditions require a special layout.

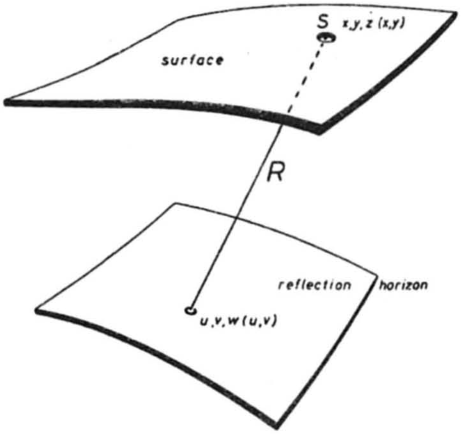

Fig. 2. Illustrating the notation used in the single layer case.

The problem of constructing the depth contour map from the corresponding map of normal depths, without constructing cross sections and taking into account refraction in space, will be solved accurately for the case of a constant overburden velocity. It will be shown that this problem can be solved by determining the point of intersection of a circle with a straight line.

Let the points on the surface of the earth (more precisely the surface of construction) be defined by the coordinates x, y, z (x, y), the x, y-plan may be sea level (Fig. 2). Let the reflecting interface be represented by the coordinates u, v, w (u, v), (u being the x-coordinate, v the y-coordinate, and w the z-coordinate): For every point x, y, z (x, y) of the earth's surface half the length of the seismic path is given by the equation

*) The normal depth is obtained by multiplying half the observed normal reflection time by the velocity.

$$R = \sqrt{(u\text{-}x)^2 + (v\text{-}y)^2 + (w\text{-}z)^2}$$

when the overburden velocity is constant. This ray must be normal to the reflecting interface.

The reflecting interface can be considered as being built up of the end points (u, v, w) of all these normal rays. Consider a section of a sphere whose radius is the normal ray and whose centre is the shotpoint. Then the reflecting interface will be tangential to the surface of this sphere. Consequently, the reflecting interface must be the envelope of all these spherical sections and can be determined according to the general theory of envelopes. The equation of the spherical sections can be written as

$$H\ (u, v, w;\ x, y) = (u\text{-}x)^2 + (v\text{-}y)^2 + (w\text{-}z\ (x, y)\ )^2 - R^2\ (x, y) = 0$$

where x and y are the parameters defining the position of the centre. Differentiating this equation with respect to x and y, and equating the resulting derivatives to zero, we obtain

$$-\frac{1}{2}\frac{\partial H}{\partial x} = u\text{-}x + (w\text{-}z)\frac{\partial z}{\partial x} + R\frac{\partial R}{\partial x} = 0$$

$$-\frac{1}{2}\frac{\partial H}{\partial y} = v\text{-}y + (w\text{-}z)\frac{\partial z}{\partial y} + R\frac{\partial R}{\partial y} = 0$$

Writing

$$u\text{-}x = X;\ v\text{-}y = Y;\ w\text{-}z = Z$$

the equations become

$$X^2 + Y^2 + Z^2 = R^2$$

$$X = -\frac{\partial z}{\partial x}Z - R\frac{\partial R}{\partial x}$$

$$Y = -\frac{\partial z}{\partial y}Z - R\frac{\partial R}{\partial y}$$

We will have to determine the common point of intersection of the sphere and the two planes defined by these equations.

The equations may also be written in the form

$$\left(\frac{X}{R}\right)^2 + \left(\frac{Y}{R}\right)^2 + \left(\frac{Z}{R}\right)^2 = 1$$

$$\frac{X}{R} = -\frac{\partial z}{\partial x}\frac{Z}{R} - \frac{\partial R}{\partial x}$$

$$\frac{Y}{R} = -\frac{\partial z}{\partial y}\frac{Z}{R} - \frac{\partial R}{\partial y}$$

This means that two planes must be made to intersect and the resulting straight line must be made to intersect the unit sphere.

In practice if the coordinate system is chosen so that the x axis is tangent to the topographic contour lines, then

$$x = 0;\ y = 0;\ \frac{\partial z}{\partial x} = 0,$$

and

$$\frac{X}{R} = -\frac{\partial R}{\partial x}$$

thus the circle

$$\left(\frac{Y}{R}\right)^2 + \left(\frac{Z}{R}\right)^2 = 1 - \left(\frac{\partial R}{\partial x}\right)^2$$

has to be cut by the straight line

$$\frac{Y}{R} = -\frac{\partial z}{\partial y}\frac{Z}{R} - \frac{\partial R}{\partial y}$$

It is possible to solve this set of equations by means of simple mechanical or electrical devices*, each of which can solve this problem and find the required envelope without the use of graphs and the attendant uncertainties caused by interpolation.

In the case of a single reflector its position can be determined immediately from the above equations, as apart from the required X Y Z all the variables can be measured.

R, $\frac{\partial R}{\partial x}$, and $\frac{\partial R}{\partial y}$ can be taken from the map of normal depths, z and $\frac{\partial z}{\partial y}$ from the topographical contour map.

If in a two-layer problem the first reflector is assumed to be a refracting interface, the solution is carried out in two stages:

1) After the normal reflection times corresponding to the second reflector have been multiplied by the velocity of the first sequence of layers and a map of normal depths has been drawn using these values, the three-dimensional position of the individual rays can determined as described above.

It is now easy to determine the points of intersection of these rays with the first reflecting horizon, by using the depth contour map of the first horizon.

2) Then the first reflecting horizon can be regarded as the new earth's surface (surface of construction) and the residual rays multiplied by the ratio of the average velocity of the first layers to the average velocity of the second layers can be regarded as new rays from the new earth's surface. (Fig. 3.)

*) Patent has been applied for by *Prakla* in respect of these devices which have been constructed by the authors in cooperation with Dr.-Ing. habil *Vetterlein* and *von Jezierski*.

The two layer problem is thus reduced to a single layer problem with an arbitrarily curved ground surface.

Similarly the multiple layer case can be reduced to the single layer case.

The method developed by Baumgarte (1955) with sectionally plane surfaces and described as "Modification of Ficticious Echo Ray Paths", is a special case of the general problem of refraction at arbitrarily curved surfaces just

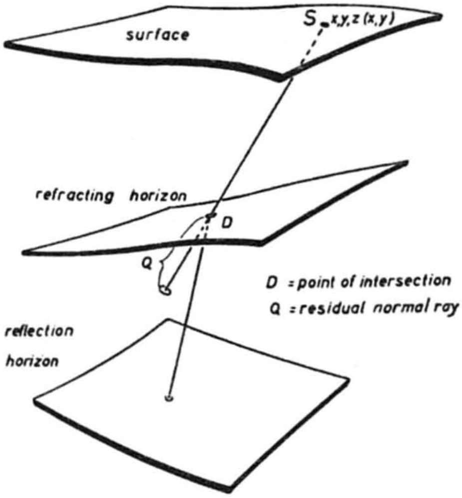

Fig. 3. Illustrating the two layer case

described. The method described makes it possible to construct exact depth contour maps of arbitrarily curved reflectors from the results of seismic reflection surveys. Cross sections can be made from these maps in any positions considered useful from a geological point of view. The method also helps to solve, with the minimum amount of work, any problem arising in practice.

An extension of the method to any sectionally continuous velocity of the individual sequence of layers is in preparation.

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