

SLANT STACK MIGRATION

BY

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Dear reader of this article!

Are you vexed by the theory of migration and does wavefield extrapolation drive you mad? If not then you are one of the lucky geophysicists. Personally, I am becoming more and more confused by the undiminishing research into the theory of migration and by the flood of publications that is produced on this subject.

Rather than give up in desperation, I persuade myself that this development is only natural. I think that the theory of migration presently shows signs of being overkilled just like many other previously successful methods.

Maybe too many of the excellent theoreticians are too young to fully appreciate the fact that in practice mother nature treats us geophysicists mostly like a wicked old bitch, having little respect for those mathematical formulae that we often find so beautiful.

Of course, I cannot dispute the need for adding refinements to an exploration tool like migration in order to apply it to more complex subsurface models and wave propagation processes. But the big question remains, which of these refinements will really tickle mother nature and cause her to give away more of her little secrets.

From experience I have realized that if I try to trick her with algorithms which exceed a certain degree of complexity, she will usually respond with a pack of lies. But perhaps I am her only victim.

Whatever the outcome of all present research into migration, it will certainly have one side-effect. Namely the method will, especially for those of us who should really know most about it – like the seismic interpreters, become less transparent and more difficult to understand. I am also worried that some of us will even be tempted to forget that the fundamental principles of migration are very simple.

There are many interesting facets of migration, most of which have been illuminated in classical papers. Most surprising to me has always been the fact that so many different kinds of manipulations on seismic reflection data can eventually lead to almost identical results, which are given to us in form of the migrated section. This indicates the fact that constructive and destructive interference phenomena are among the most puzzling and fascinating properties that can be associated with the wave equation.

Such phenomena can also be well observed in *slant stack migration*. This is a new migration procedure which I would like to introduce in the following text. It is, in my opinion, conceptionally the simplest migration scheme that exists for seismic sections. In order to conceive the scheme one only requires a limited understanding of the wave propagation phenomena.

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The scheme that is subsequently described is valid for the migration of CDP stack sections in the presence of a constant velocity medium. It can however be easily extended to media with dipping velocity boundaries and three-dimensional media.

In order to understand slant stack migration no more knowledge about wave propagation is required than an appreciation of the facts that

- A) in a homogeneous medium, plane waves of arbitrary pulse forms will propagate in any direction without change of shape. They travel perpendicular to the wavefront with the propagation velocity of the medium, and
- B) many such plane waves can, by superposition, create some specific wavefield.

Concerning these two statements: plane waves, although the simplest solutions to the homogeneous wave equation, are in their infinite extension in space – and maybe time – strictly speaking only *mathematical concepts*. In other words they cannot physically be realized. But this does not imply that the superposition of many or even an infinite number of plane waves could not describe, say, the wavefield that is caused by some real and possibly complex explosion mechanism.

Plane waves can in this sense be used for a somewhat similar purpose as harmonic oscillations. In their full temporal (or spatial) extension these are also not physically realizable. But is there a geophysicist who does not remember from his first elementary course in Fourier analysis that such “mathematical concepts” can, by superposition, create a realizable pulse of some finite length?

At this point it should be mentioned that in order to decompose an arbitrary wavefield into plane waves one might also have to consider so-called inhomogeneous (or evanescent) plane waves, but these can be justifiably ignored in the following.

Before describing slant stack migration, let me make the familiar assumption that a CDP stack section can be approximately described by a wavefield that can be thought to originate at time zero in so-called “exploding reflectors” and which moves up to the seismic line with half the actual propagation velocity.

It is difficult to visualize the upgoing wavefield of the exploding reflector model as many or an infinite number of plane waves that emerge along the seismic line with various wave shapes and at various times and at a variety of emergence angles. But once we have accepted this fact, then we can say that each plane wave will have its response concealed in the recorded wavefield, i.e. the CDP stack section. In other words the CDP stack section can be thought of as being the superposition of individual plane wave responses.

Slant stack migration can therefore be conceived as consisting of

1. Detecting each plane wave response in the CDP stack section.
2. Using this to construct, by a simple mapping process, the upgoing plane wave at time zero in the depth domain, i.e. the plane wave that caused this particular response.
3. Summing all upgoing plane waves at time zero in the depth domain with the result of obtaining the migrated section.

In order to extract a plane wave response from a CDP stack section one could apply a “very narrow velocity pass filter” to it. This would filter out the responses that are caused by *all* plane waves that emerge for the specified emergence angle or apparent velocity.

The velocity filter just mentioned is practically realized by nothing other than a “slant stack” applied to the traces of the CDP stack section. A slant stack – also known as a “delay sum” – is performed by delaying (or advancing) the equidistantly spaced traces of the section successively by a constant amount prior to summing them all together.

Once the plane wave responses for a selected emergence angle (or apparent velocity) have been filtered out of the CDP stack section it is easy to construct the “plane wave image” for these responses. It consists of the upgoing plane waves which have caused these particular responses at zero time in the depth domain.

The wavefronts of plane waves emerging with the angle α (Figure 1) satisfy the equation $t = (x - x_0) \tan \beta / v$ in the time domain and $z = (x - x_0) \tan \alpha$ in the depth domain. Please note that the relationship $\tan \beta = \sin \alpha$ holds, where v is the medium velocity.

So the converting of plane wave responses into the related plane wave image is achieved simply by a linear vertical scaling conversion of the responses from time to depth. In other words the values of the plane wave responses that were obtained along the constant moveout trajectories of the $(x - t)$ domain are mapped onto the respective moveout trajectories of the $(x - z)$ domain (see Figure 1).

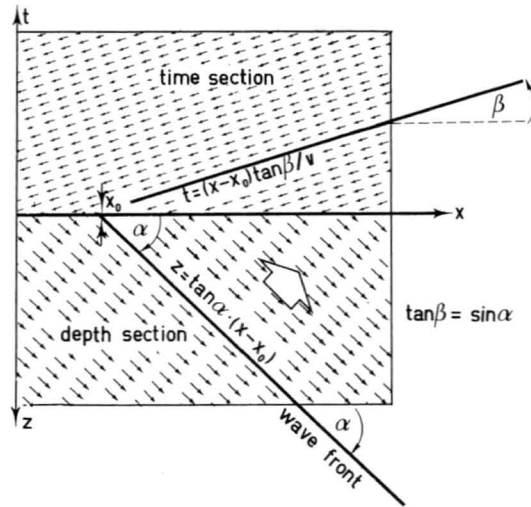


Fig. 1 Construction of plane wave image:

The value that is obtained by summing all amplitudes of the CDP stack “time section” along the line $t = (x - x_0) \tan \beta$ is placed on all points of the line $z = (x - x_0) \tan \alpha$.

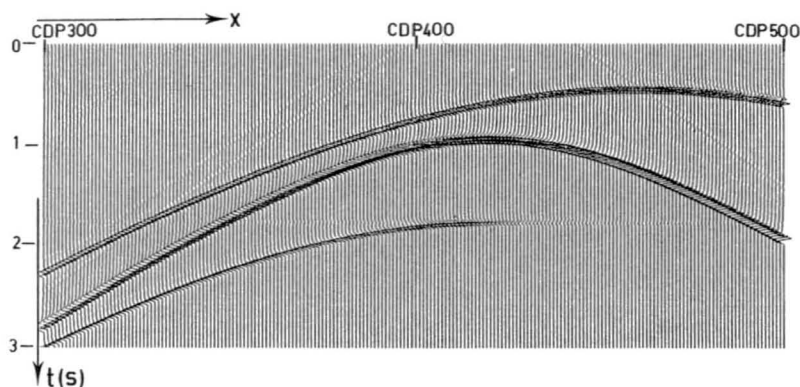


Fig. 2a CDP stack section of 10 km width (trace spacing 50 m).

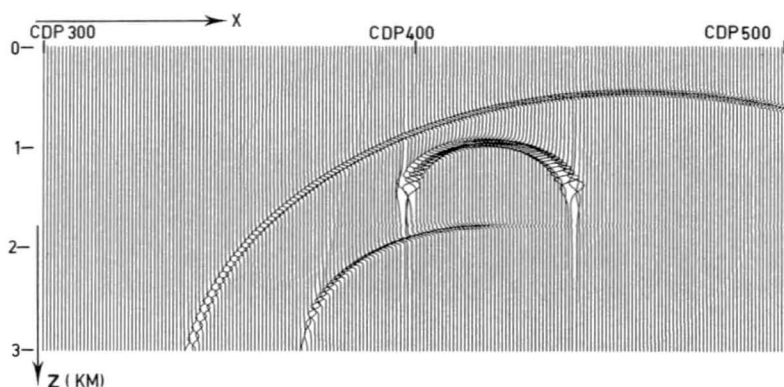


Fig. 2b Depth section.

The slant stack migration procedure will now be explained with the help of an example. Figure 2a shows a CDP stack section that belongs to the depth section of Figure 2b. The depth section consists of a constant velocity medium ($v = 2000$ m/s) in which three “exploding reflectors” are shown. The wavefield “leaving” each reflector is described by wavelets of different shapes and laterally varying amplitudes.

Figure 3 shows the responses of plane waves that were filtered out of the CDP stack section for the emergence angle $\alpha = 0$. The filtering operation was done by first per-

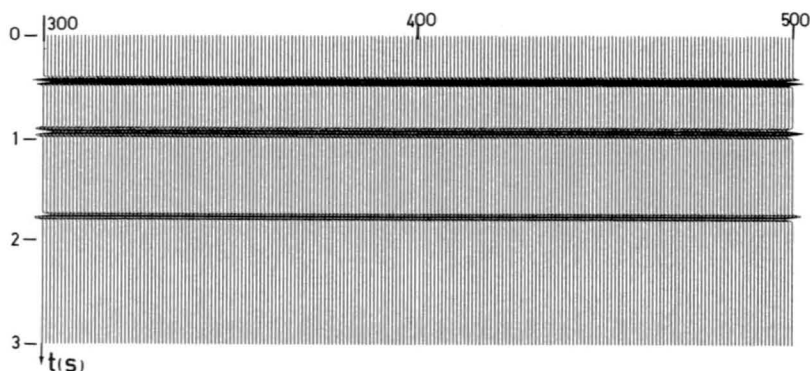


Fig. 3 Plane wave responses filtered from CDP stack section for $\alpha = 0$.

forming a horizontal addition of all traces of the CDP stack section and secondly *spreading out* the resulting “slant stacked trace” back onto all the traces. Finding the plane wave image for these responses now only involves changing the t-axis of Figure 3 into a z-axis using the relationship $z = v \cdot t/2$.

Next, those plane wave responses were filtered out of the CDP stack section of Figure 2a which relate to the emergence angle $\alpha = -45^\circ$. The filtered responses are shown in Figure 4. Their plane wave image was *added* to the one obtained for $\alpha = 0$. The result is shown in Figure 5.

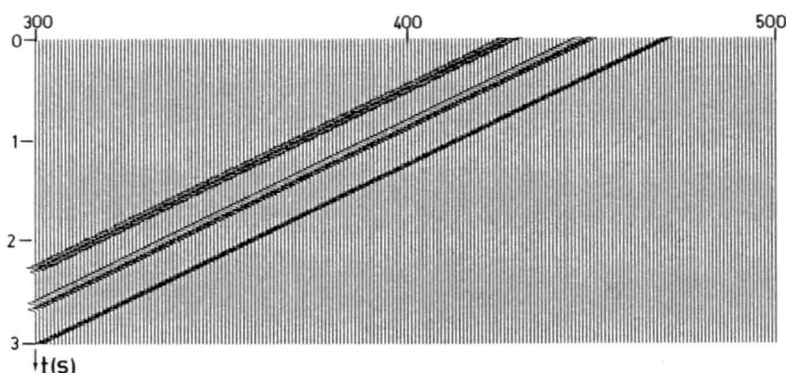


Fig. 4 Plane wave responses filtered from CDP stack section for $\alpha = -45^\circ$.

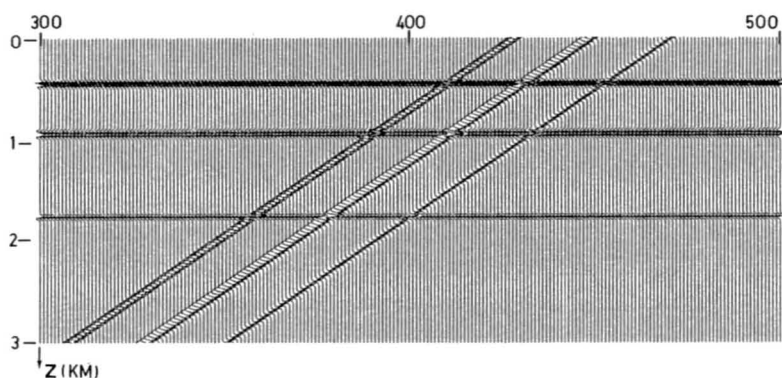


Fig. 5 Sum of plane wave images for $\alpha = 0$ and $\alpha = -45^\circ$.

The filtered response of Figure 4 was again obtained by (a) slant stacking all traces of the CDP stack section of Figure 2a along those trajectories that belong to $\alpha = -45^\circ$ and (b) spreading the slant stacked trace back onto all the traces along the trajectories that entered the slant stack.

In another experiment, plane wave responses were extracted with the above slant stack procedure for all emergence angles $\alpha = -90^\circ, -85^\circ, \dots, -10^\circ, -5^\circ, 0^\circ$ ($\Delta\alpha = 5^\circ$), and the sum of all plane wave images was obtained. The result is shown in Figure 6. We obtain our first indication which shows we are on the right approach to recovering the “migrated section” of Figure 2b. The three expected reflectors can already be vaguely recognized among some strongly organized noise.

To resolve the migrated section better, the experiment was repeated for the angles $\alpha = -90^\circ, -88^\circ, -86^\circ, \dots, 0^\circ, \dots, 86^\circ, 88^\circ, 90^\circ$ ($\Delta\alpha = 2^\circ$). The migrated section is now more clearly visible than with $\Delta\alpha = 5^\circ$. However, it was not before using all angles between $\alpha = -90^\circ$ and $\alpha = +90^\circ$, with incremental steps of $\Delta\alpha < 1^\circ$, that a result comparable to the one shown in Figure 2b was obtained.

You may have already noticed that the described procedure did not produce boundary effects where the horizons meet the margins of the section. These boundary effects do, of course, exist like in any other migration scheme, however, they were suppressed for the sake of keeping this presentation simple.

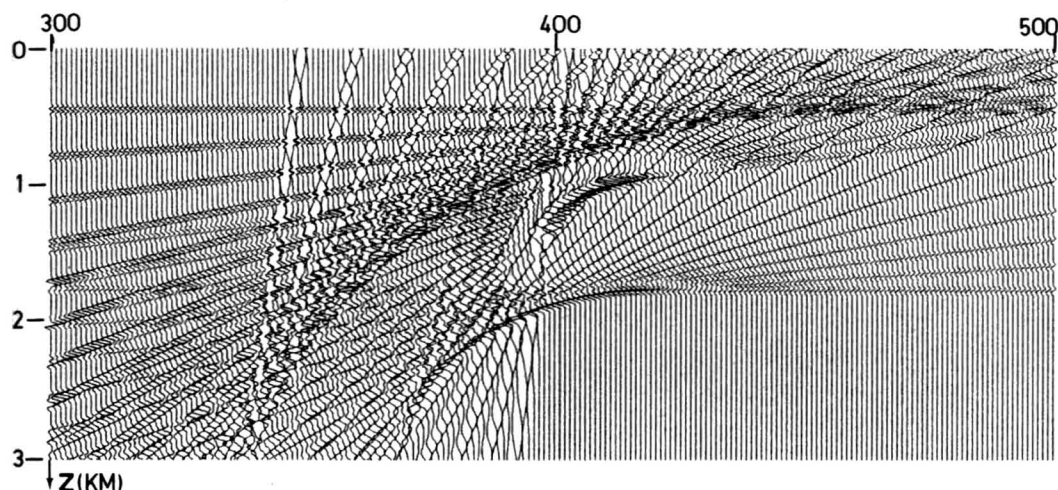


Fig. 6 Sum of plane wave images for $\alpha = -90^\circ, -85^\circ, \dots, -5^\circ, 0^\circ$ ($\Delta\alpha = 5^\circ$).

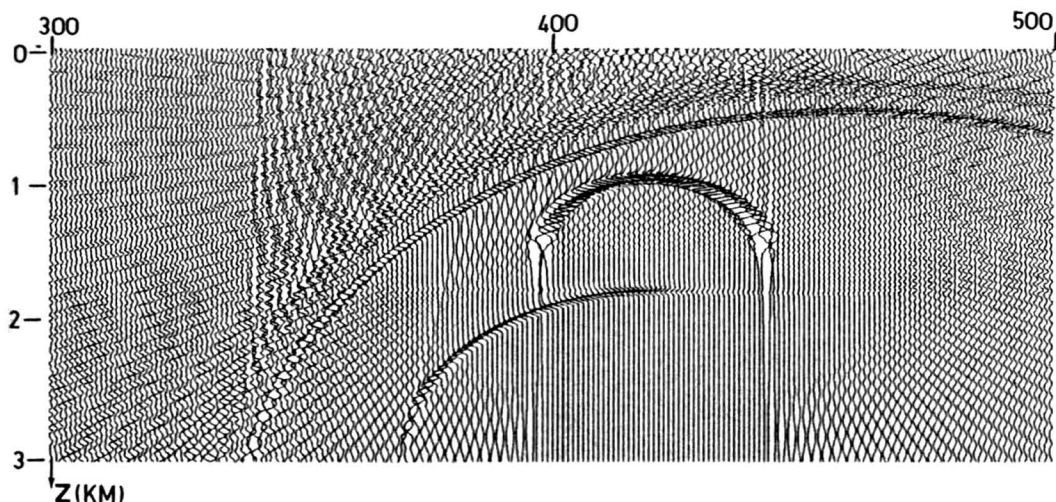


Fig. 7 Approximately recovered migrated section.

This completes the description of slant stack migration. One only needs a little imagination to see how the method can be extended from two to three dimensions. It is also rather easy to invert the scheme and model a CDP stack section from a depth section. In such a case one would start by slant stacking the depth section.

Even the extension of the theory to a medium with a plane dipping velocity boundary is easily conceived. Each plane wave image that is constructed from the filtered plane wave responses need then only be bent along the given velocity boundary at depths according to Snell's law.

Before concluding this paper I would like to point out that slant stack migration is, naturally, closely related to all other migration schemes. In particular it is easy to establish its relationship to $(f-k)$ migration. Taking the $(f-k)$ transform of a CDP stack section implies *de facto* also a decomposition of the section into plane harmonic waves. It can be shown that $(f-k)$ migration may be interpreted as doing the same kind of operations as described above on harmonic plane wave components. They are, however, done implicitly in the frequency-wave number domain and not explicitly in the time-space domain as is used in slant stack migration.

The similarity of both methods does not mean that the results will be exactly the same. For instance, I believe the problem of reducing noise can be better approached in slant stack migration than in any other scheme.

ACKNOWLEDGEMENTS

I have always enjoyed the inspiring discussions on various topics which I had with Professor Th. Krey, and I wish him many more years of good health during which our profession can continue to benefit from his tremendous experience and his ingenious ideas. Personally, I am looking forward to many more talks with him.

Regarding this particular paper I would also like to express my sincere thanks to Paul Gutowski and Sven Treitel. I have no doubt that the origins of the ideas that have been mentioned in this paper go back to those many tea-break discussions I had with both of them in the AMOCO canteen in Tulsa, Oklahoma during 1979.

PRAKLA-SEISMOS' invitation to contribute to this special issue presented me with the excellent opportunity of writing a paper and having it printed without having to worry about the usual criticism of those almighty anonymous reviewers of our professional journals.

Had I submitted this paper for publication in one of the journals I would have considered myself lucky if the paper was not rejected and returned to me after 6 to 12 months with those all too familiar recommendations that the paper should (a) be substantially rewritten or (b) be resubmitted as a short note, (c) be sent to another reviewer (where it would disappear for another 6 to 12 months) or (d) include a practical example, and so on. I would of course be prepared for all this, but I pity those young researchers who are unaware of this procedure and have to contact these anonymous men for the first time.