PRAKLA-SEISMOS INFORMATION No. 31

2-D/3-D Wave Equation Migration

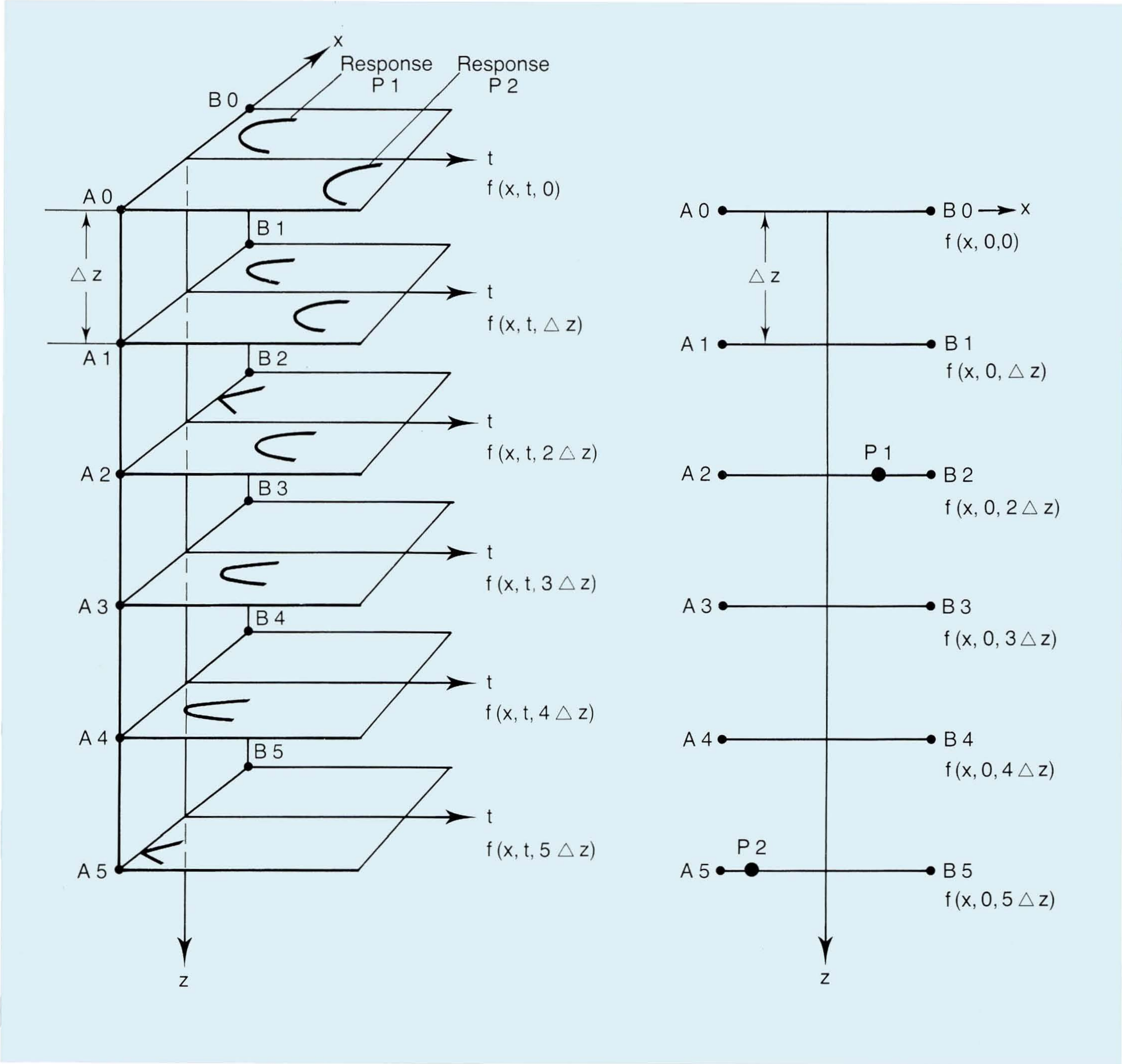


Fig. 1: Downward-Continuation and Imaging

Contents

Seismic time sections are graphic displays of recorded traveltimes. They can only be a rough approach to geological sections and are difficult to interpret in case of complex tectonics. Dipping reflectors will be shifted, synclinals may appear as anticlinals and diffractions superimpose to give confusing events. To correct for these distortion effects and to improve lateral resolution is the goal of migration.

The elastic subsurface is considered to be a quasi-acoustic medium. The propagation of acoustic waves is mathematically controlled by the acoustic wave equation. Hence, neglecting shear waves and converted waves, this partial differential equation can be used to determine the wave field at any depth z_i from surface data. This process is called downward continuation. A measure for the reflection coefficient at depth z_i is obtained by selecting the amplitudes for zero-travel time and for zero-offset. This process is called imaging. Thus migration of seismic data generally consists of two procedures:

- downward continuation
- imaging.

This principle is illustrated for 2-D zero-offset data in Fig. 1, where, for simplicity, the downward continuation of two hyperbolic response curves from two single diffractors at different depths is considered.

There are generally different possibilities to implement the downward continuation process with the help of the acoustic wave equation. Mainly from the economic point of view in seismic data processing and to overcome mathematical difficulties, a simplified form of the acoustic wave equation is used for migration, the so called "one-way"-wave equation. This equation describes only primary reflections and does not consider multiples.

Essentially **four approaches,** generating different migration operators, have been developed and improved for routine application:

- **KIRCHHOFF Summation approach (x,t domain)**
- **Finite Difference approach (x,t domain)**
- **Frequency Wavenumber approach (f,k domain)**
- **Frequency Space approach (f,x domain)**

Concerning their algorithms the four techniques are completely different, each matching distinct problems. They are recursive and/or non-recursive. They are different in their sensitivity to lateral velocity variation, to the amount of dip to be handled and to the character of data themselves concerning frequency content and signal/noise ratio. From that point of view it may be concluded that one should not rely only on one particular migration technique.

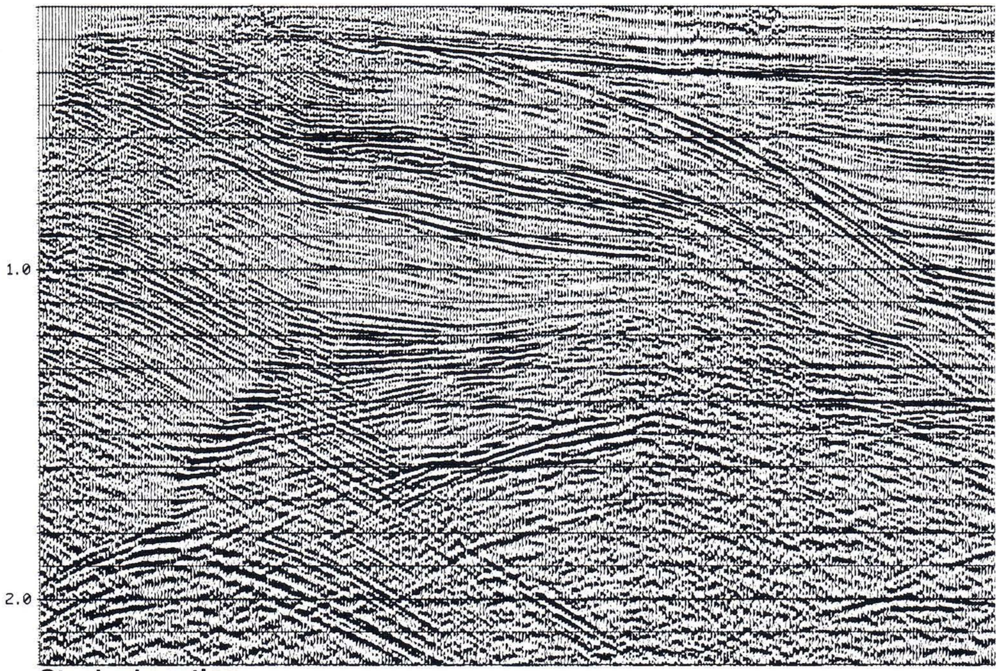
The conventional way to apply migration schemes on seismic data is to start from CDP-stacked traces **(migration after stack = MAS).**

This however implies that the stacking procedure is a good approach to the generation of zero-offset traces. But in many cases it is not, when the reflection surfaces are strongly curved or steeply dipping. Then the reconstruction of subsurfaces structures is carried out directly from the surface data characterized by a non-zero distance between the emitters and receivers. This process is called **"migration before stack" (MBS).**

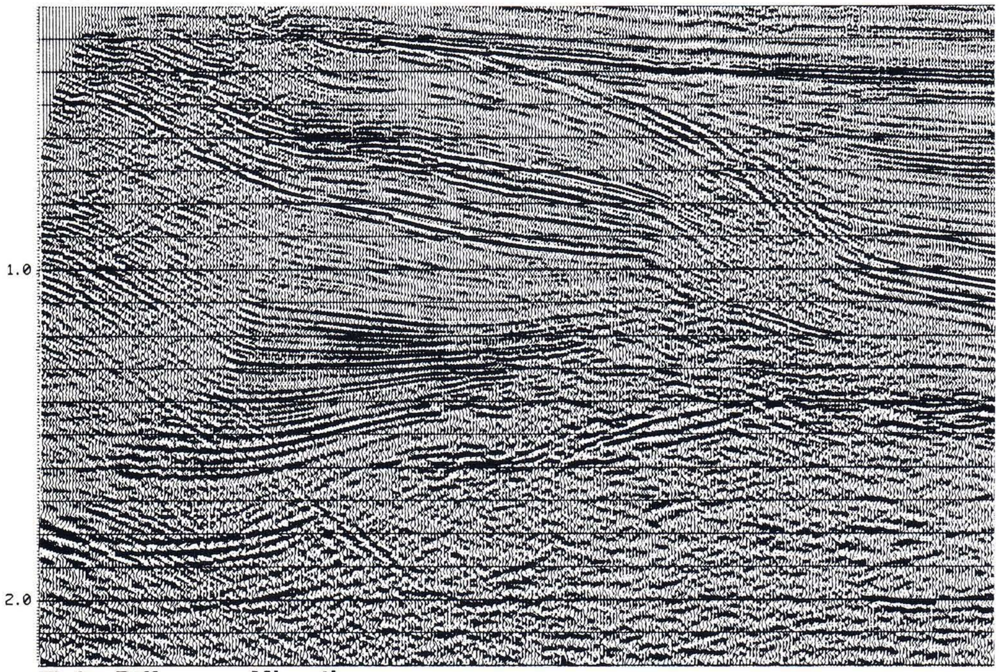
An important point for the effectiveness of the migration process is the velocity model to be applied. In general velocities vary with depth and along the survey line. Thus ray bending effects of incident and reflected rays at the velocity interfaces will be incorporated in seismic reflections. This phenomenon is only incompletely considered in conventional migration techniques, but is well considered in the procedure of **depth migration.**

The seismic reflection method is in general a 3-dimensional problem. In consequence 2-dimensionally surveyed and processed lines cannot result in reliable sections of the subsurface because they contain interference from diffraction events from outside the vertical plane. The 3-D method has now become of age in geophysical exploration. The extension of many algorithms for application to 3-D data has promoted **3-D migration** to the central procedure in any 3-D data processing sequence as a step forward to delivering true geological sections.

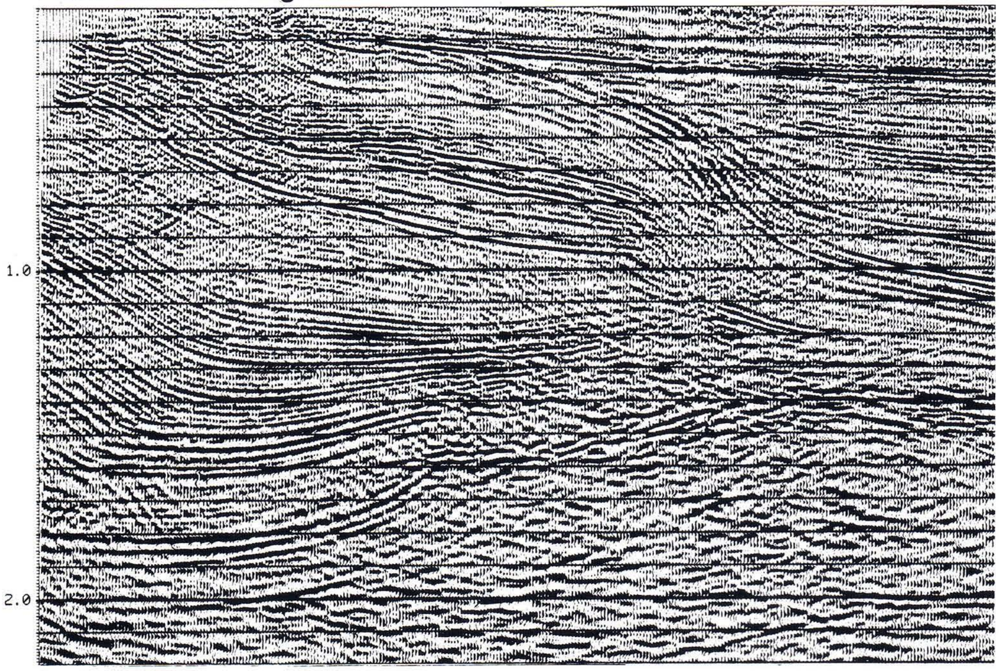
Migration after Stack (MAS)



Stacked section



Finite Difference Migration



f,x Migration

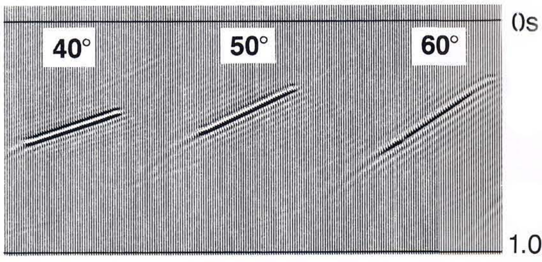
Whenever reliable velocities can be estimated to stack seismic data according to the common reflection point assumption, migration will be applied after stack. Stacking improves the signal/noise ratio and reduces the amount of data drastically, thus subsequent migration is less expensive than migration before stack.

Amplitudes and reflection character of the stacked data are widely preserved in all MAS-methods. Thus, the type of data with respect to frequency content, steepness of dip, signal to noise ratio and lateral velocity variation decides which migration scheme may be applicable.

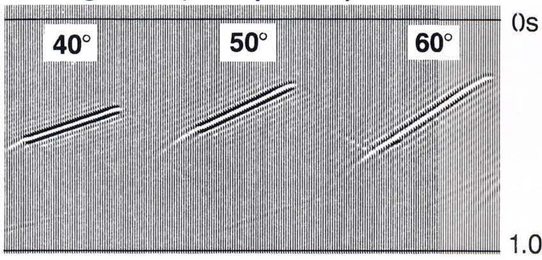
Well known procedures for routine migration processing have been up to now the Kirchhoff method, the Finite-Difference method and the f,k method.

A new tool which perhaps offers the most flexible treatment of wave field extrapolation is the frequency-space method. Lateral and vertical velocity variation can be sufficiently considered and steeper dips will be well resolved. A frequency dependent optimization of the downward continuation operator can easily be incorporated. However, data dependent dispersion effects and wrap-around noise may sometimes interfere with the success of f,x migration. There are also "higher order" operators available improving lateral resolution and attenuating dispersion effects, as can be seen in comparison of the 45°-operator and the 55°-operator applied to synthetic data.

f,x Migration (45°-operator)



f,x Migration (55°-operator)

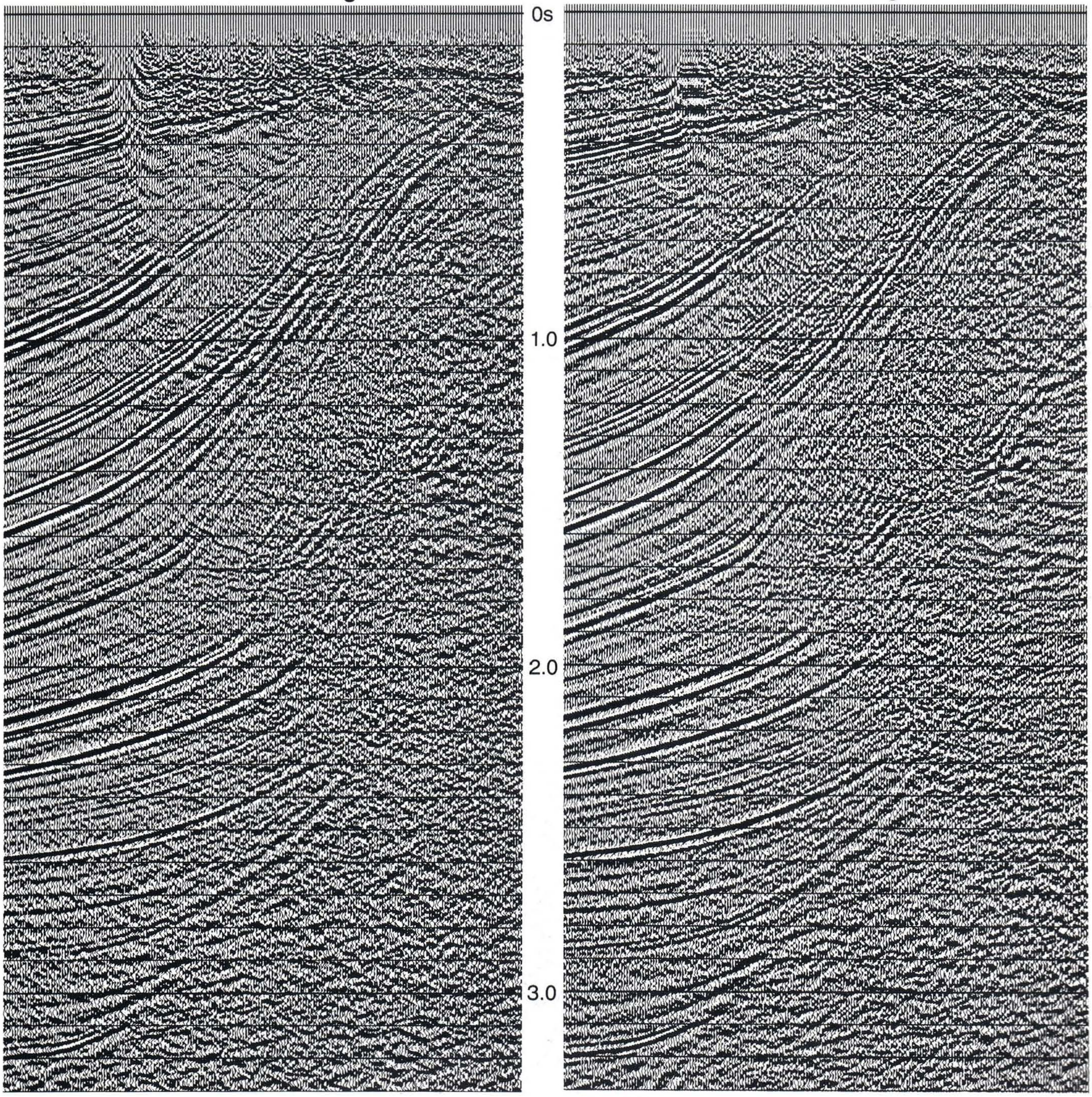


Often difficulties arise in the derivation of optimum stacking velocities from data of complex subsurface structures with significant lateral velocity variations. Where dipping and curvature of reflectors are encountered the CDP-philosophy of different source-receiver separations must fail. Also, when recording seismic data in drillholes, it has to be realized that CMP-stacking is no longer an effective tool for imaging the subsurface even in the case of horizontal layering.

The only way to overcome these problems is migration before stack, applied either in the original source-receiver coordinate system or the CDP-offset coordinate system, which is usually preferable. To reduce the computer intensive migration of each individual common-offset profile, seismic traces are gathered to produce partial stacks of small offset ranges. Velocities used for migration of each common offset section are adapted to the corresponding offset range. Individually migrated partial stacks are then stacked using velocities determined from a standard analysis routine.

The improvement of this migration approach can clearly be realized in steep dip regions with extreme flexures or in faulted areas.

Finite Difference Migration after stack | **Finite Difference Migration before stack**



Depth-Migration

Migration results are often presented in the time scale in order to preserve the character of stacked data for a detailed comparison. However, according to the goal of migration, namely to produce true geological sections, the only reasonable display domain can be the depth.

True depth section on one hand means correct positioning of all reflection events, correct positioning on the other hand means consideration of ray bending according to Snell's law.

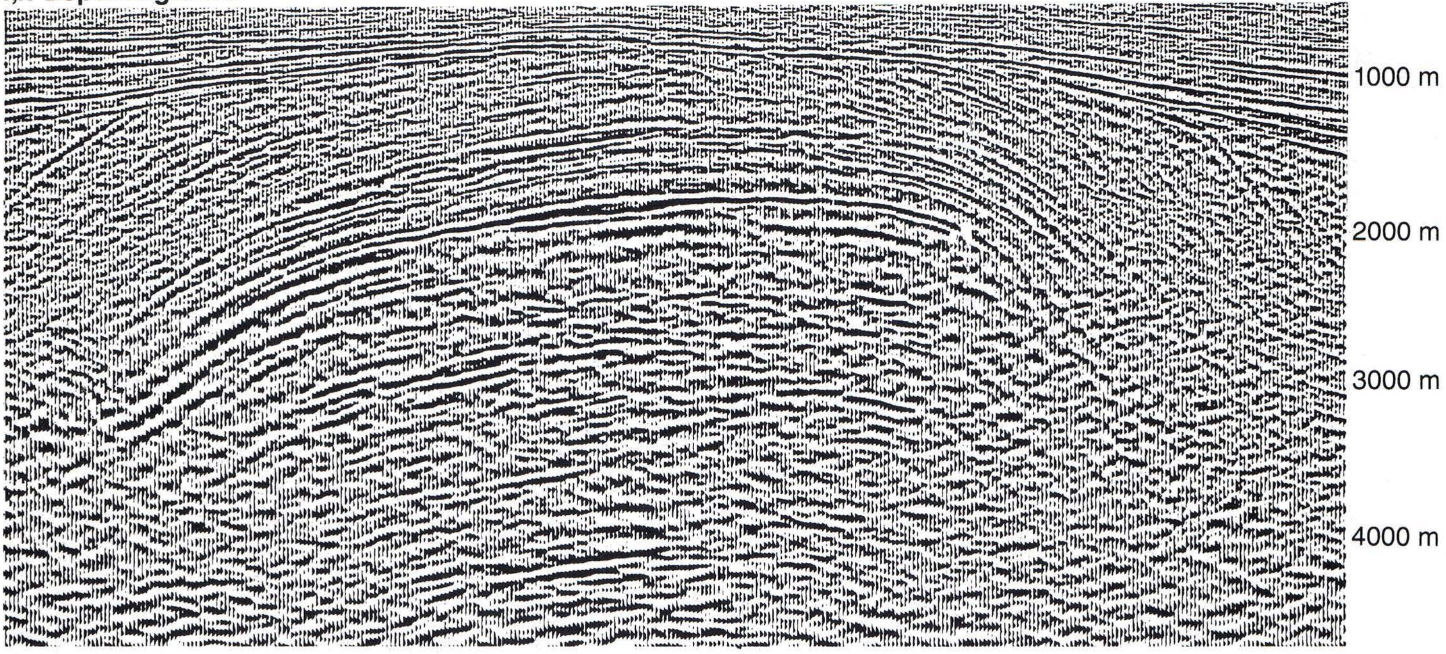
Up to some years ago this weak point of conventional migration techniques could only be overcome by ray tracing procedures carried out on pre-interpreted reference horizons. Even today these ray migration results are a valuable aid to the interpreter.

When Hubral 1976 proposed the application of the so called image ray correction to time migrated data the term "depth migration" was introduced as synonym for considering ray bending in the presence of large velocity contrasts.

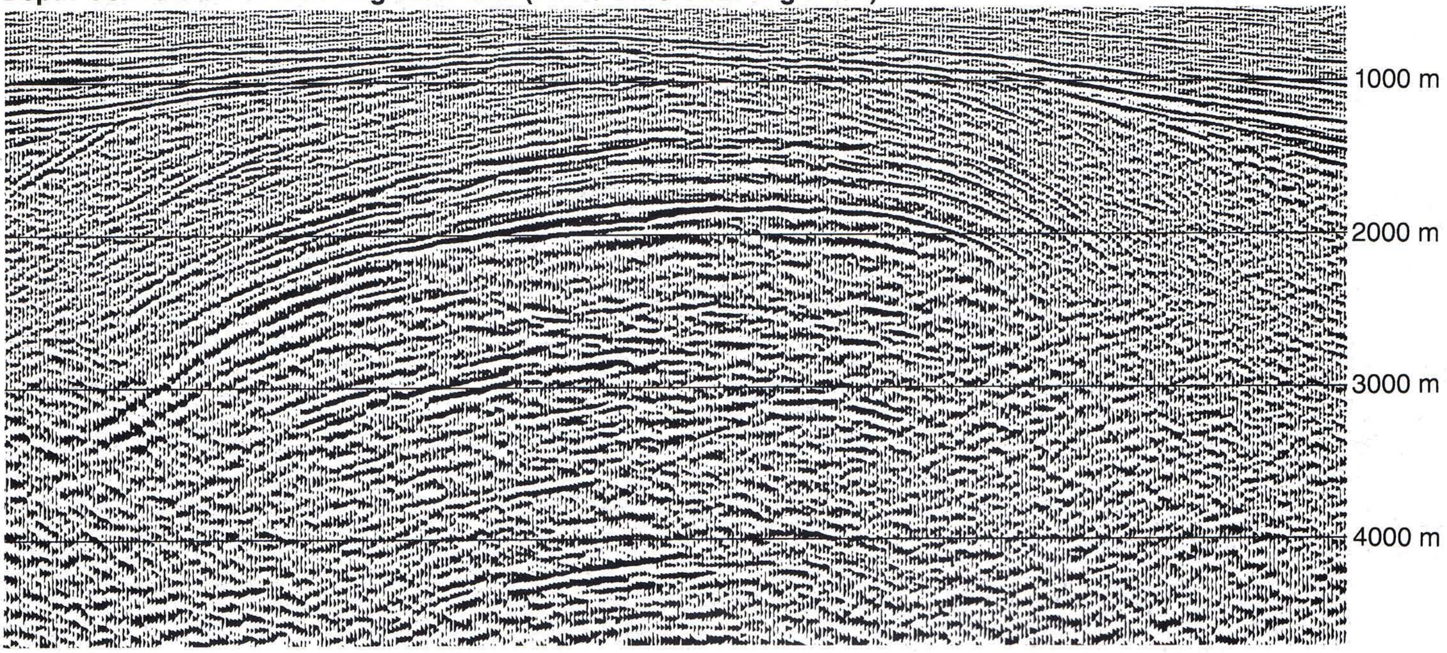
Many approaches have been developed since to be incorporated into standard migration schemes. The presentation of fully migrated data directly in the depth domain can e.g. be realized by the f,x method (see example).

It should be emphasized that especially in presenting true depth sections the proper treatment of the velocities used for migration is an indispensable prerequisite.

f,x Depth Migration



Depth Conversion of time migrated data (Finite Difference Migration)



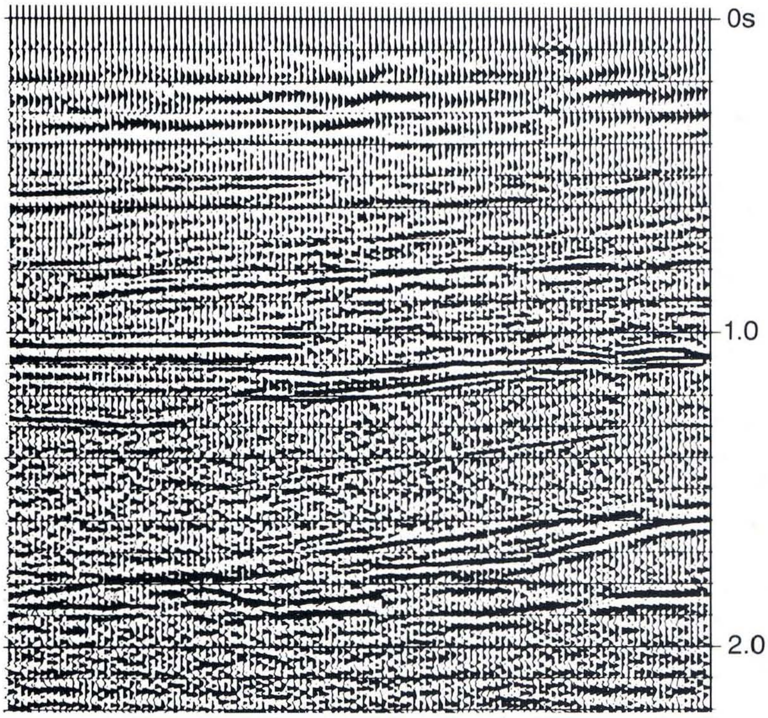
The extension of the 2-D migration to a 3-D algorithm was first implemented by the KIRCHHOFF summation technique. This procedure, which is still a suitable method for migrating single lines of a 3-D survey, had however proved to be uneconomical for the migration of the whole 3-D data volume. Mainly from this aspect two distinct 3-D migration schemes have been developed for routine application, both are based on the same idea but differ in their implementation:

1. **3-D migration as splitting approach**
2. **3-D migration as two-step approach.**

The splitting principle approximates the 3-D downward continuation operator by 2-D operators acting in perpendicular directions. They are applied sequentially deriving the wave field for all depth levels. The imaging process is then performed by extracting the amplitudes at $t = 0$ from the 3-D downward continued wave field. The wave field extrapolation is carried out recursively using finite difference algorithms.

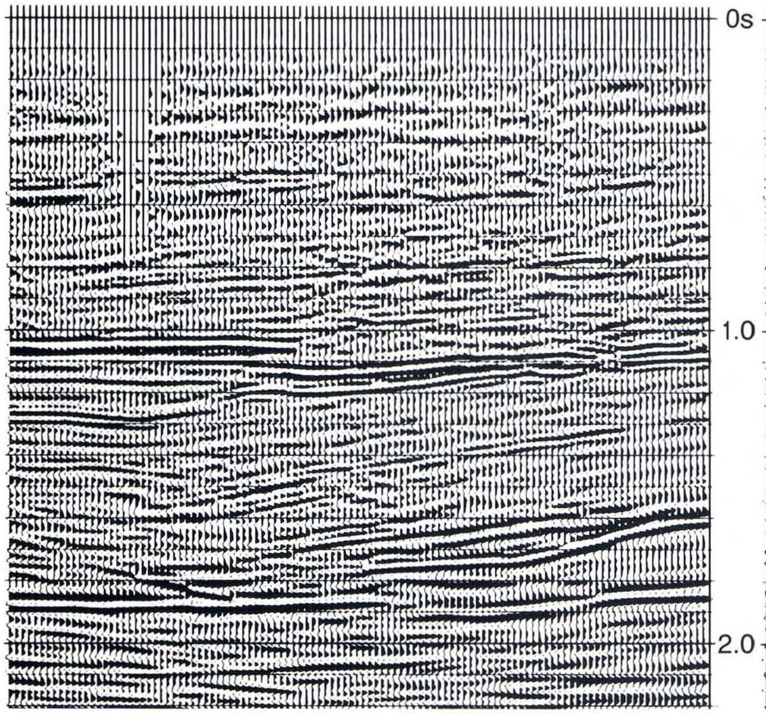
The two-step method of the 3-D migration process is based on the full separability of the 3-D problem. Thus the 3-D migration operator can be replaced by the application of two 2-D operators, performing downward continuation and imaging for all depth levels separately in two perpendicular directions. All existing 2-D migration methods and their combinations are applicable as shown in the example. 3-D splitting migration is the more accurate procedure from the aspect of better handling of lateral velocity variations.

3-D migrated sections provide the best starting point for a reliable interpretation enabling a profound insight into the subsurface.

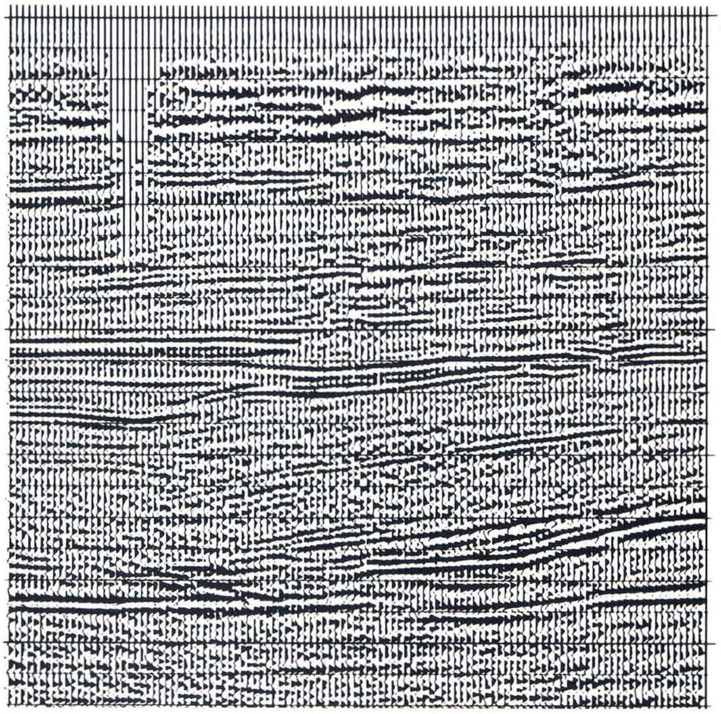


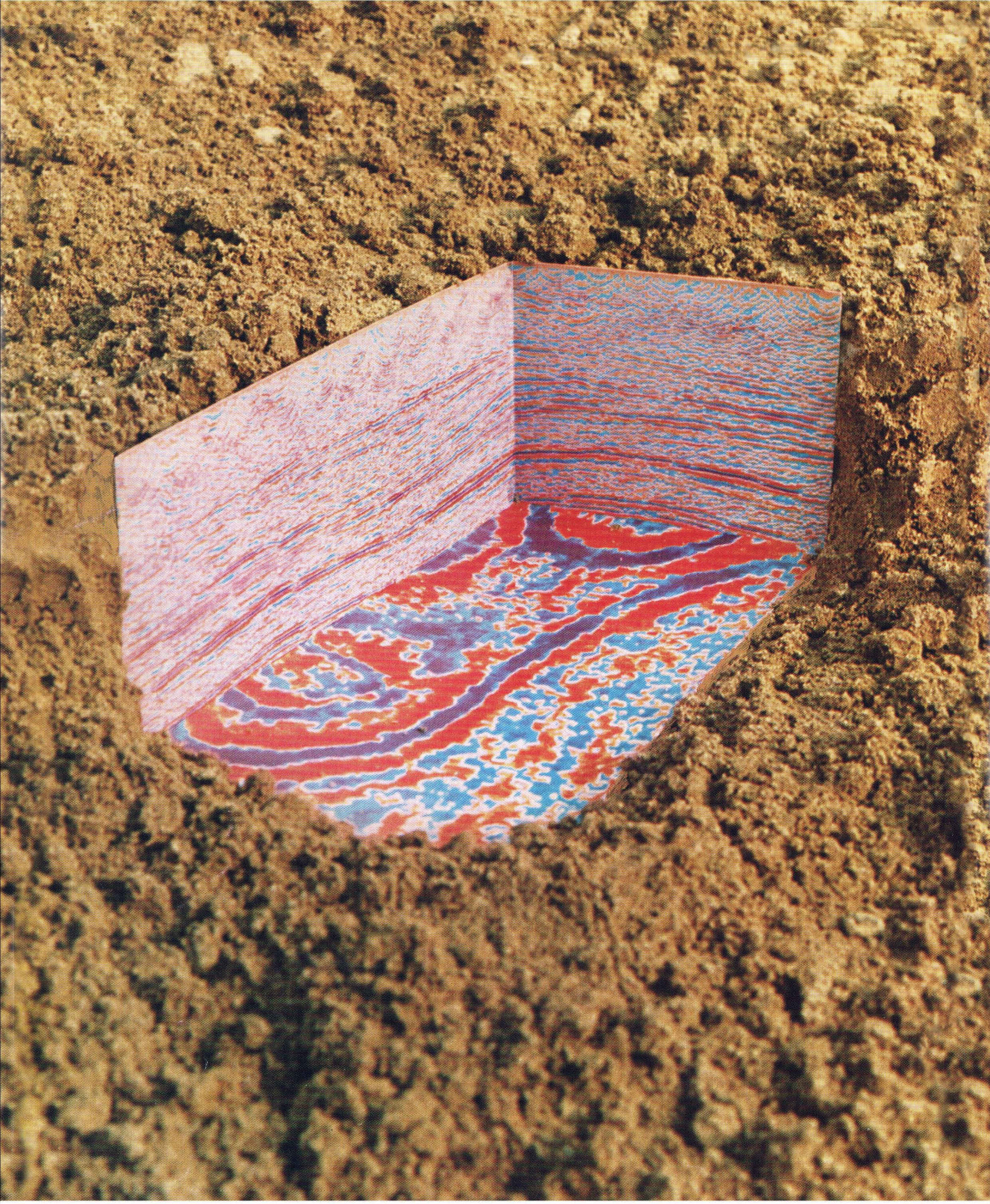
a. 3-D Finite Difference Migration (splitting method)

b. 3-D Kirchhoff Migration (two-step method)



c. Combined Two-Step Migration
1. Kirchhoff Migration
2. Finite Difference Migration





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